Dynamic Allocation Strategies
for Absolute and Relative Loss Control

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Abstract

Dynamically allocating wealth between cash and a performance-seeking portfolio allows investors to limit drawdowns to a chosen pre-defined level or risk-budget. This paper introduces variants and generalizations of the maximum drawdown metric and the corresponding risk-control strategies, such as excess drawdown, relative drawdown, and rolling drawdowns control. We discuss the relevance of these risk-management constraints, prove that they can be insured using dynamic allocation, and characterize the corresponding risk-control strategies.

Keywords: Risk Management, Portfolio Insurance, Hedging Overlay, Loss Aversion, Benchmarks. JEL classification: G11, G110.

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Introduction

Investment strategies controlling *drawdowns* (cumulative losses over intermediate horizons) are popular in investment management. Empirical observations show that funds yielding overly poor performance often face significant withdrawal/redemption, or liquidation, leading managers to decrease leverage following poor performance to increase the fund’s survival likelihood, which implies a drawdown control motive (see Goetzmann, Ingersoll, and Ross, 2003; Pangeas and Westerfield, 2009; Lan, Wang, and Yang, 2013). Interestingly, Guasoni and Obloj (2011) show that the optimal strategy of a manager compensated with common high-water-mark based performance fees coincides with the portfolio of a hypothetical investor maximizing utility from the fund assets (instead of fees) but facing the drawdown constraint, i.e., maintaining the fund value above a given percentage of its running maximum at all times. Furthermore, drawdowns control has been shown to be consistent with rational choice theory (Kim, 2011; Roche, 2013), and with *loss aversion*, which is the central feature of prospect theory (Kahneman and Tversky, 1979, 1991).

The Time-Invariant Portfolio Protection (TIPP) or drawdown control strategy was introduced by Estep and Kritzman (1988), and Grossman and Zhou (1993) showed that the strategy is utility maximizing under the drawdown constraint. Cvitanic and Karatzas (1995) generalized the optimal solution for the case with multiple performance-seeking assets and a stochastic short-term interest rate, and Elie and Touzi (2008) extend it to the case with intermediate consumption\(^1\).

Following a dynamic allocation rule between the locally riskless asset (cash) and a performance-seeking asset, the strategy limits the portfolio’s maximum drawdown to a given tolerance level set ex-ante. However, lower interest rates and lower loss tolerance levels increase the opportunity cost of the maximum drawdown constraint. In practice, holding a large proportion of cash is inconvenient, or even contractually banned for professional investment managers. Furthermore, holding an important share of cash with the purpose of controlling short-term losses can conflict with other risk-management objectives, such as hedging long-term liabilities.

This paper introduces a variation of the drawdown control strategy compatible with this

\[^1\]Other investment strategies using drawdowns as the risk metric of concern include Carr, Zhang, and Hadjiliadis (2011), and Zhang, Leung, and Hadjiliadis (2013), who study max drawdown-based insurance contracts; Chekhlov, Uryasev, and Zabarankin (2000, 2005) introduce a family of risk measures called Conditional Drawdown-at-Risk (CDaR) and solve a mean return-CDaR optimization. Portfolio sensitivities to max drawdown are studied in Pospisil and Vecer (2010) and probabilistic properties of drawdowns in Douady, Shiryaev, and Yor (2000), Magdon-Ismail, Atiya, Pratap, and Abu-Mostafa (2004), Pospisil and Vecer (2008), Hadjiliadis and Večer (2006), Pospisil, Vecer, and Hadjiliadis (2009), and Zhang and Hadjiliadis (2010, 2012).
need or will to avoid using cash as the reserve asset. The strategy controls the drawdown in excess of the drawdown experienced by a given reserve asset, the latter being any chosen safe enough asset. This alternative loss-control strategy presents a lower opportunity cost than the standard max drawdown strategy through a higher maximum allocation to the performance-seeking asset. Furthermore, by using a reserve asset different from cash, such as a liability hedging portfolio or a bond portfolio with relatively small drawdowns, the strategy makes the popular short-term loss control motive compatible with long-term performance objectives.

This paper also discusses how the risk-metric and drawdown-control strategy can be adapted to a relative performance context in which the objective is to control the underperformance of a given (investable) benchmark instead of absolute losses. We introduce the concept of relative drawdown\(^2\), which is a measure of maximum cumulative underperformance with respect to a given benchmark, as well as the dynamic allocation strategy to control this relative risk. Unlike (excess) absolute drawdowns, relative drawdowns may happen during ‘bull markets’. This type of investment strategy can be of particular interest for investors entering ‘alternative betas’ and/or active investment funds, for which the expected outperformance usually comes with an important benchmark underperformance risk.

Besides this, we introduce dynamic allocation strategies that control excess drawdowns and relative drawdowns measured over a rolling period of time, instead of since inception, which allows a reduction of the opportunity cost of the strategies. An important characteristic of all the strategies introduced hereafter is that they insure their corresponding risk-management objectives for investors entering at any point in time into the strategy. Thus, unlike other portfolio insurance strategies only offering their guarantee to investors entering the strategy at initial date, they are adapted for implementation to open funds.

The risk-control strategies studied here can insure that its corresponding performance constraints are respected by maintaining the portfolio value above a Floor value process at all times. In this paper we formalize a golden rule for this type of portfolio insurance strategies, which is a sufficient condition to prove that a given Floor process and corresponding performance constraint can actually be insured using standard dynamic asset allocation-based portfolio insurance strategies. We show that the strategies introduced in this paper respect that rule. Furthermore, we also provide an estimate of the upper bound of the multiplier process of this type of strategies, which is the maximal multiplier that allows these

\(^2\)Not to be confused with the standard maximum drawdown (MDD) concept, which is sometimes called relative MDD to differentiate it from its version expressed in dollar terms as opposed to relative change terms (returns). In this article we always work with drawdowns expressed as returns.
strategies to respect its constraint under discrete-time trading, in the general case with a locally risky reserve asset.

1 Dynamic Allocation Based Portfolio Insurance

Portfolio insurance strategies such as Constant Proportion Portfolio Insurance (CPPI) (Perold, 1986; Black and Jones, 1987; Black and Perold, 1992; Perold and Sharpe, 1995) and TIPP guarantee that the portfolio respects a given performance constraint by following an asset allocation rule that prevents the value of the portfolio, denoted $A$, to fall below a Floor value $F$. This kind of strategies splits wealth between a risky, performance-seeking asset $S$, and a reserve or safe asset, $B$. The standard allocation rule consists of maintaining at every time $t$ a proportion of wealth invested in asset $S$ as follows:

$$\omega_S(t) = m_t \times \left(1 - \frac{F(t)}{A(t)}\right),$$

where $m_t > 0$ is a constant or an adapted time-varying process, and the remaining wealth is invested in the reserve asset. Whenever $A$ approaches $F$, wealth is reallocated towards the reserve asset to prevent the portfolio from breaching its Floor value. For such a strategy to be able to insure the performance constraint $A(t) \geq F(t)$ for all $t$, it is sufficient that the reserve asset super-replicates the Floor process, i.e., $\frac{dB(t)}{B(t)} \geq \frac{dF(t)}{F(t)}$ at all times (assuming continuous-time trading and prices). Hereafter we provide a less restrictive requirement, which is a sufficient (although not necessary) condition to check that a given Floor value process is insurable using dynamic allocation.

Let $t_0$ denote the initial investment date, unless otherwise stated. Provided that $A(t_0) > F(t_0)$, a type (1) strategy can insure that the corresponding value process $A(t) \geq F(t)$ for all $t > t_0$, if its reserve asset $B$ and Floor satisfy the following conditions for all $s \leq t$:

1. $\frac{B(t)}{B(s)} \geq \frac{F(t)}{F(s)} \quad \forall \ 0 \leq s \leq t \text{ whenever } C.1 := \frac{F(t)}{F(s)} > \frac{A(t)}{A(s)}$,

or if at time $t$,

2. $F(t) \leq A(t)$ by definition of the Floor process. We refer to these two conditions as the golden rule (G.R.) of type (1) portfolio insurance strategies, which state that the reserve asset should replicate or super-replicate the Floor only in periods where the portfolio’s return is lower than the Floor’s relative change. Indeed, the super-replication condition is not necessary in periods when the distance to the Floor is increasing, or at times

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4Equivalently, the dollar allocation to $S$ is $w_S(t) = m \times (A(t) - F(t))$. 

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when the Floor is lower than portfolio value by definition. For instance, while for the CPPI
the Floor value has the same dynamics as the reserve asset at all times by the definition of
its Floor process, i.e. \( dB(t)/B(t) = dF(t)/F(t) \) for all \( t > t_0 \), for the max drawdown control
strategy the reserve asset must super-replicate the Floor (equation 9 below) only during
drawdown periods of \( A \); indeed, in “bull market” periods in which \( A \) increases its distance
to the Floor, the reserve asset of the strategy (cash) underperforms the Floor. Nonetheless,
the max drawdown constraint is also insurable if cash is used as reserve asset. In appendix
B we prove the sufficiency of the golden rule, assuming that the multiplier process remains
below its discrete-time trading upper bound (the latter is detailed in appendix A).

The allocation of type (1) strategies, such as the CPPI, the TIPP and the strategies that
will be introduced hereafter would theoretically change continuously over time. However,
in practice, trading happens in discrete time, and asset prices may present “jumps”. In
appendix A we derive the upper bound that the multiplier of type (1) strategies needs to
respect, so that \( A(t) \) remains above \( F(t) \) even with discrete time trading and price jumps.

In the CPPI literature, the reserve asset is often assumed to be a locally riskless as-
set paying a constant rate of return considered “relatively small” compared to the worst
possible loss in the risky asset. For this reason the upper bound of the multiplier is often
approximated as the inverse of the expected shortfall of the risky asset. However, when the
reserve asset \( B \) is locally risky, the right tail of its return distribution also becomes relevant
to estimate the upper bound of the multiplier. In appendix A we show that a conservative
estimate of the upper bound of the multiplier of a type (1) strategy is,

\[
\hat{m}_t = \frac{1}{LT_S(t) - RT_B(t)}, \tag{3}
\]

where \( LT_S(t) \) and \( RT_B(t) \) are left and right tail estimates of the (conditional) return distribu-
tions of asset \( S \) and \( B \) respectively.

Although the CPPI and the TIPP are both type (1) strategies, they have very dis-
tinct behaviours because their Floor formulas, and thus the risk-management constraints
they serve are very different. Indeed, while the TIPP limits losses incurred over any inter-
mediate horizon, the CPPI strategy only limits the losses experienced by the portfolio as
measured with respect to the initial date. As a consequence, the TIPP can guarantee its
risk-management objective for investors entering at any point in time into an investment
fund, while CPPI only offers its guarantee to an investor entering the strategy at initial
date\(^5\).

The maximum drawdown (MDD) of an investment is defined as the largest value loss

\(^5\)In effect, the CPPI can present severe cumulative losses over arbitrarily long intermediate horizons, as
the portfolio loses former cumulated capital gains during drawdown periods of the risky asset.
from a peak to a bottom observed at current time \( t \). More precisely, for a value process \( A \), the drawdown at time \( s \), denoted \( D_s(A) \), is the percentage loss experienced by \( A \) with respect to its running maximum observed since time \( t_0 \), denoted \( M_{t_0}^A(s) \), attained for the last time at \( s_{t_0,A}^* \). Therefore, the maximum drawdown observed since inception \( t_0 = 0 \) to current time \( t \), denoted by \( \bar{D}_{t_0,t}(A) \), is defined as follows:

\[
\bar{D}_{t_0,t}(A) := \sup_{t_0 \leq s \leq t} D_s(A)
\]

(4)

where \( D_s(A) := -R_A(s_{t_0,A}^*, s) \)

(5)

\[
M_{t_0}^A(s) := \sup_{t_0 \leq q \leq s} \{ A_q, M_{t_0}^A(t_0) \}
\]

(6)

and \( s_{t_0,A}^* := \sup_{t_0 \leq q \leq s} \{ q : A_q \geq M_{t_0}^A(s) \} \),

(7)

where \( R_A(t_1, t_2) \) denotes the simple return of \( A \) between the two instants \( t_1 \leq t_2 \). The maximum drawdown constraint on portfolio \( A \) is

\[
\bar{D}_{t_0,t}(A) \leq x, \quad \forall \ t \in [t_0, \infty),
\]

(8)

where the risk budget \( x \in (0,1) \), represents the maximum percentage loss of the current capital the investor is willing to tolerate at any point in time.

In order to define a type (1) dynamic allocation strategy that can insure a performance constraint such as (8), one needs to
\( i) \) define the Floor process, \( ii) \) prove that if \( A_t > F_t \) for \( t \in [t_0, \infty) \) then \( A \) satisfies the constraint, and \( iii) \) prove that the Floor and reserve asset satisfy the golden rule (2).

For the MDD strategy, the Floor value is defined at every time \( t \) as follows

\[
F(t) = k \sup_{s \in [t_0, t]} A(s),
\]

(9)

where \( k := (1 - x) \). If the value of the portfolio is always above the MDD Floor (9), then constraint (8) is respected, since the following statements are equivalent,

\[
A(s) \geq F(s) = kA(s_{t_0,A}^*),
\]

(10)

\[-R_A(s_{t_0,A}^*, s) \leq 1 - k = x\]

(11)

for all \( s \in [t_0, t] \) and \( t \in [s, \infty) \). Notice that the MDD Floor (9) is a strictly non-decreasing function of time. Consequently for the strategy to satisfy the golden rule (2), the reserve asset must be an asset with a strictly positive performance at all times to (super-)replicate the Floor; hence bounded to be cash. Grossman and Zhou (1993) and Cvitanic and Karatzas (1995) consider a similar strategy for which the drawdown and the corresponding Floor are
defined in terms of a high water mark (HWM) or discounted wealth process. In that case, the Floor value is equal to

\[ F(t) = k S^0(t) \sup_{s \in [t_0, t]} \frac{A(s)}{S^0(s)}, \]

where \( S^0 \) is discount value process growing at rate \( r \) (e.g. a savings account paying a short-term interest rate). This Floor presents a more binding constraint than (9), since the HWM grows exponentially at the short-term rate instead of being flat during drawdown periods of \( A \). Lan et al. (2013) and Goetzmann et al. (2003) point out that the fund’s HWM should be adjusted downward when money flows out of the fund due to management fees or if investors continuously redeem capital at a consumption rate \( y \). Hence the HWM grows at the rate \( r - y \) and for \( y = r \), the discount process \( S^0 \) is constant, the HWM is equal to the running maximum \( M^A_{t_0} \), and the HWM Floor (12) equals (9).

**Remark 1** Unlike the CPPI, for a given multiplier value \( m \), the drawdown control strategy has a maximum exposure to the risky asset equal to

\[ \sup_{s \in [t_0, t]} \omega_S(s) = m \times x, \]

which is reached every time the portfolio’s value attains a new maximum. As a consequence, this kind of strategy has a less pronounced trend-following behavior than the CPPI strategy\(^6\).

**Remark 2** The maximum drawdown strategy insures a proportion \( k \) of the maximal capital gains made by the portfolio since time \( t_0 \). To see this, divide equation (10) by \( A(t_0) \),

\[ \frac{A(s)}{A(t_0)} \geq k \frac{A(s^*_{t_0}, \cdot)}{A(t_0)} \quad \forall s \in [t_0, t] \]

\[ \Leftrightarrow 1 + R_A(t_0, s) \geq k(1 + R_A(t_0, s^*_{t_0}, \cdot)) \]

(14)

for all \( t_0 \in [0, s] \) and \( s \in [t_0, \infty) \). This is the so-called ratchet effect on wealth.

The G.R. and the drawdown constraint impose investors to hold a proportion of cash so as to insure the max drawdown constraint. This implies an opportunity cost to investors seeking to get exposure to the performance asset \( S \). In order to decrease the opportunity cost of the strategy, one may modify the constraint by restricting the max drawdown measurement to a rolling look-back period of time \( p > 0 \) instead of calculating it since inception\(^7\).

\(^6\)While the TIPP exposure reaches an upper bound every time the value of the portfolio attains a new maximum, the CPPI’s risk exposure grows exponentially during ‘bull’ markets and remains theoretically unlimited.

\(^7\)From there on, we refer sometimes to the maximum drawdown as the “global MDD”, and to the maximum drawdown with a restricted calculation horizon as the “rolling MDD”.

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The corresponding risk control strategy was recently introduced by Yang and Zhong (2012), who study its optimal multiplier. Hereafter, we uncover some interesting properties of this strategy.

The rolling MDD metric and Floor are obtained by setting $t_0 = (t - p)^+$ in equations (4), (5), (6), (7), (9), and (12). This changes the risk-control objective: instead of controlling the maximum drawdown over the life of the investment, the strategy limits the drawdowns observed over a given rolling period of time with respect to the rolling running maximum $M_{t-p}^A$. Thus, by imposing a less binding constraint, the global maximum drawdown of the portfolio can in fact be higher than the risk budget.

On the other hand, by excluding former maximum values, the running rolling maximum of the strategy can actually decrease over time, implying a higher exposure to the risky asset, everything else equal, and hence potentially decreasing the opportunity cost of the strategy.

**Remark 3** The rolling drawdown strategy avoids being fully invested in cash for an indefinite amount of time in case of Floor violation. In effect, the maximum time period during which the strategy ends up fully invested in cash, in the case of a gap event at time $t_{t_0,A}^{gap} > t_{t_0,A}^\ast$, is a period equal to $p - (t_{t_0,A}^{gap} - t_{t_0,A}^\ast) < p$, where $t_{t_0,A}^{gap} := \inf_{t_0 \leq s \leq t} \{s : A_s < F_s\}$.

**Remark 4** Unlike the global MDD strategy, the rolling drawdown strategy can provide a guarantee on the portfolio value net of management fees, even for a management fee rate $y > r$, by anticipating the fees to come over the horizon $p$, where $r$ is the risk-free rate. Hence, the risk budget is changed to

$$\hat{x} = x - y_p,$$

where $y_p = (1 + y)^p \Delta t - 1$, $y$ is the annual management fee rate, and $\Delta t$ is the time step, e.g., $\Delta t = \frac{1}{12}$ for monthly time steps, and $p$ is expressed in number of months.

**Corollary 1** Let the trailing performance of portfolio $A$ be defined as its return over a period of length $p$, i.e., $R_A(t - p, t)$ for all $t \in [p, \infty)$. A portfolio defined by a type (1) investment strategy with a rolling drawdown Floor satisfies for $t \in [p, \infty)$

$$R_A(t - p, t) \geq -x.$$

**Proof.** The result follows from equation (14) with $t_0 = (t - p)^+$, noticing that $R_A(t_0, t_{t_0,A}^\ast) \geq 0$. Thus, a rolling drawdown strategy can guarantee that the trailing performance of the portfolio stays above $-x$ at all times\(^8\).

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\(^8\)Notice that the trailing performance is defined only for $t \geq p$. 
Remark 5 A guarantee of a positive return over the investment period $t_0$ to $t$ (and on the trailing performance for $t_0 = (t - p)^+$) can be offered to the investor, i.e., $R_A(t_0, t) > 0$, whenever the maximal realized return of the portfolio over the (rolling) period attains the following level:

$$R_A(t_0, t^*_0, A) > \frac{x}{1 - x}.$$  

(16)

This follows from inequality (14) (with $t_0 = (t - p)^+$ for the trailing performance).

Moreover, notice that if the maximum drawdown of the risky asset lasts for a period $p$, the Floor value will start to decrease when the risky asset starts to recover. Therefore, the allocation to the risky asset will be relatively higher (with respect to the global MDD strategy) during the recovery period of $S$. This property is in line with a conjecture by Yang and Zhong (2012) who assert that, “a drawdown look-back period $H$ somewhat shorter than or similar to the market decline cycle is the key to achieve optimality.”

2 Controlling Excess Drawdowns

The dynamic allocation strategies controlling the maximum drawdown previously discussed may require holding an important amount of cash in the portfolio for relatively long time periods. This implies an important opportunity cost for investors, particularly in low interest rate environments or for relatively small values of $x$. For instance, according to a recent report from Blackrock, in the US over the period 1926-2012, unlike stocks and bonds, cash yielded a negative compounded return after adjusting for inflation and taxes, as measured by the 30-day US Treasury bill index. This suggests that holding a important proportion of cash for short-term loss control purposes can pose a hurdle to long-term performance. Furthermore, in practice holding an large share of cash is sometimes even banned for professional investment managers, as “large investors in a specific fund stipulate that the fund avoid holding cash” (Alster, the New York Times, October 5, 2013).

Furthermore, holding an important share of cash with the purpose of controlling short-term losses can conflict with other risk-management objectives, such as hedging the long-term commitments of long-term investors, such as pension funds, for whom the “risky-free” asset with respect to their long-term liabilities is a portfolio of bonds with long duration (see for instance Martellini and Milhau, 2009). In order to address this need to avoid holding important proportions of cash for short-term loss controlling purpose, in what follows, we introduce a type (1) dynamic allocation strategy that limits the drawdown of the portfolio in excess of the drawdown experienced by any given stochastic investable reserve asset. Furthermore, the allocation to the performance-seeking asset of this strategy has a higher upper
bound than the allocation of a MDD strategy with the same risk budget and multiplier, which also implies a lower opportunity cost.

**Definition** The excess drawdown of portfolio $A$ with respect to reserve asset $B$ at time $s$ is defined as

$$ED_s(A,B) := D_s(A) - D_s(B);$$

the maximum excess drawdown at time $t$ is defined as

$$ED_{t_0,t}(A,B) := \sup_{s \in [t_0,t]} ED_s(A,B),$$

while the excess maximum drawdown is defined as

$$EMDD_{t_0,t}(A,B) := D_{t_0,t}(A) - D_{t_0,t}(B).$$

**Definition** (EDD Floor): Let the Excess Drawdown Floor value process for a type (1) strategy be defined as

$$F(t) = k \frac{A(t^*_{t_0,A})}{B(t^*_{t_0,A})} B(t),$$

for all $t \in [t_0, \infty)$, where $A$ is the value of the portfolio, $B$ the value of the reserve asset.

Notice that $t^*_{t_0,A}$ in (17) is defined with respect to $A$ and not to the ratio $\frac{A}{B}$. In Appendix H, we show that this Floor respects the G.R.; thus it is insurable using a type (1) asset allocation strategy.

The following proposition shows that if the value of the portfolio is always above the EDD Floor (17), then its maximum excess drawdown is lower than the risk budget $x$. Furthermore, it shows that the difference in maximum drawdowns (thus the excess maximum drawdown) is also lower than the risk budget, and it provides an upper bound for the maximum drawdown of the portfolio.

**Proposition 1** Let $A$ denote the value of the portfolio and $B$ denote the value of the reserve asset. If the value of portfolio $A$ is above the corresponding EDD Floor (17) at all times, then $A$ and $B$ satisfy the following conditions:

$$EMDD_{t_0,t}(A,B) \leq ED_{t_0,t}(A,B) \leq x \quad (18)$$

$$if R_B(t^*_{t_0,A}, t) \leq 0 \Rightarrow D_{t_0,t}(A) \leq x + D_{t_0,t}(B) \quad (19)$$

$$if R_B(t^*_{t_0,A}, t) > 0 \Rightarrow D_{t_0,t}(A) \leq x. \quad (20)$$

for all $t \in [t_0, \infty)$.  


The proof of the Proposition is presented in Appendix C.

**Remark 6** The maximum drawdown of a portfolio with an EDD Floor is smaller than $x$ when $R_B(t^*_{t_0,A}, t) > 0$ and smaller than $x + |R_B(t^*_{t_0,A}, t)|$ otherwise. Thus, the maximum drawdown of the portfolio can be higher than $x$ and the excess drawdown limit can eventually be reached, i.e., $\overline{ED}(t_0, t) = x$, only if reserve asset $B$ experiences a drawdown during the max drawdown period of portfolio $A$. Given that 1) this coincidence is needed (although not sufficient) to spend all the risk budget $x$ and 2) the maximum excess drawdown is always higher than the excess maximum drawdown, then, the EDD strategy is *conservative* for an investor seeking to limit the excess maximum drawdown of the portfolio to $x$.

The next corollary shows that portfolios with an EDD Floor present a *conditional* wealth ratchet effect similar to the MDD strategy. Indeed, the EDD strategy insures a proportion $k$ of the sum of the maximal capital gains experienced by the portfolio from $t_0$ to current time $t$, and the capital gains made by the reserve asset between the last time-record of the running maximum $t^*_{t_0,A}$ and current time $t$.

**Corollary 2** A given portfolio $A$ bounded from below by an EDD Floor satisfies the following conditions:

\[
1 + R_A(t_0, t) \geq k(1 + R_A(t_0, t^*_{t_0,A}))(1 + R_B(t^*_{t_0,A}, t)), \text{ for simple returns,}
\]

\[
r_A(t_0, t) \geq r_A(t_0, t^*_{t_0,A}) + r_B(t^*_{t_0,A}, t) - \tilde{x}, \text{ for log returns,}
\]

for all $t_0 \in (0, t)$ and $t \in (t_0, \infty)$, where $\tilde{x} = -\log(1 - x)$.

The proof of the corollary is presented in Appendix D.

**Remark 7** From equation (1) and the definition of an EDD Floor, it follows that the allocation to $S$ of the EDD strategy has an upper bound given by:

\[
\omega_S(s) = m \times \left(1 - k \frac{A(t^*_{t_0,A})}{A(t)} \frac{B(t)}{B(t^*_{t_0,A})}\right) < \max\{1; m \times x\},
\]

for $t \in [t_0, \infty)$. Notice that a MDD strategy will have a lower maximal allocation to $S$ than an EDD strategy with the same $m$ and $x$, if $m \times x < 1$ (see equation 13). In general, $\omega_S > m \times x$ whenever $\frac{B(t)}{B(t^*_{t_0,A})} < \frac{A(t)}{A(t^*_{t_0,A})} \leq 1$; that is, during drawdown periods of the reserve asset, the allocation to the performance asset of the EDD can be higher than $m \times x$.

Thus the opportunity cost of an EDD strategy is likely to be lower over long horizons than the opportunity cost of a similar MDD strategy, due to its higher maximal potential allocation to the performance-seeking asset.
Remark 8 Notice that at every time the portfolio value attains a new maximum, i.e., 
\( t = t_{t_0,A}^* \), the EDD strategy exposure is 
\( \omega_S(t_{t_0,A}^*) = m \times x \). Furthermore, if the reserve asset has a positive (negative) performance between the former running maximum time-record, 
\( s_{t_0,A}^* \), and the new maximum time record \( t_{t_0,A}^* \), then the risk exposure of the strategy will increase (decrease) between 
\( s = t_{t_0,A}^* - \delta \) and 
\( t_{t_0,A}^* \) proportionally to 
\( R_B(s_{t_0,A}^*, t_{t_0,A}^* - \delta) \), for an arbitrarily small \( \delta \). Assuming that 
\( A(s) \approx A(s_{t_0,A}^*) \), then 
\[ \omega_S(t_{t_0,A}^*) - \omega_S(t_{t_0,A}^* - \delta) \approx m \times k \times R_B(s_{t_0,A}^*, t_{t_0,A}^* - \delta). \]

As shown above, the maximum drawdown of a portfolio with an EDD Floor may exceed \( x \) by an amount smaller or equal to the maximum drawdown realized by the chosen reserve asset. Thus, the investor may choose a reserve asset with either a known maximum drawdown (for instance a portfolio with a maximum drawdown Floor, in which case the total max drawdown is equal to the sum of the risk budgets of the excess and max drawdown strategies) or an asset with tolerable levels of expected maximum drawdown, e.g. short-duration bond portfolio.

In order to further decrease the opportunity cost of the strategy, a rolling window EDD Floor and constraint can be introduced. However, a naive rolling window version of the EDD constraint and Floor, similar to the rolling MDD constraint and Floor, obtained by simply setting 
\( t_0 = (t - p)^+ \) in equation (17) for the EDD Floor and in equation (18) for the constraint, would satisfy (only) condition ii) for the introduction of a portfolio insurance strategy. In effect, this naive version of the rolling EDD constraint is not insurable with a type (1) asset allocation strategy. Hereafter, we introduce an insurable rolling EDD Floor that complies with a modified rolling EDD constraint and the golden rule; thus insurable with a type (1) strategy.

Consider the alternative, less restrictive rolling EDD constraint:

\[ \bar{D}_{t-p,A}(A) - \bar{D}_{t_0,A}(B) \leq x. \]  

(22)

Definition Let the conditional rolling maximum time process be defined as

\[ t_{t-p,A}^{**} = \begin{cases} 
\sup_{s \in \mathcal{G}(t)} \{ s_{(s-p)+,A}^* \} , & \text{if } t > t_0, \text{ } M_{t-p}^A(t) < M_{s-p}^A(s) \text{ and } \mathcal{G}(t) \neq \emptyset \\
 t_{(t-p)+,A}^*, & \text{otherwise}
\end{cases}, \]

(23)

where

\[ \mathcal{G}(t) = \left\{ t_0 \leq s < t : \frac{Z(t_{(t-p)+,A}^*)}{Z(s_{(s-p)+,A}^*)} > 1 \right\} \]

(24)

9In Appendix H.2 we show that the naive rolling EDD Floor does not comply with the golden rule. Using Monte Carlo simulations (available upon request from the author) we did find violations of the naive rolling EDD Floor and constraint; thus confirming that the Floor is not insurable.

10Less restrictive with respect to the global EDD constraint (18).
and a rolling period $p > 0$, for all $t \in [t_0, \infty)$, where $Z := \frac{A}{B}$.

**Definition (Rolling EDD Floor):** Let the Rolling Excess Drawdown Floor value process for a type (1) strategy, and rolling period $p$, be defined as

$$F(t) = k\frac{A(t_{t-p,A}^{**})}{B(t_{t-p,A}^{**})}B(t), \quad (25)$$

for all $t \in [t_0, \infty)$, where $A$ is the value of the portfolio, $B$ the value of the reserve asset.

The definition of the rolling EDD is thus similar to its naive version, with the exception that its reference time for the running maximum of $A$ rolls forward unless the current rolling running maximum decreased and the condition defining the set (24) is satisfied. In effect, the conditional running maximum time process of the rolling EDD Floor given by (25) only rolls forward at times in which the golden rule cannot be violated. Hence, Floor (25) also complies with the golden rule (see details in Appendix H.2).

**Proposition 2** Let $A$ denote the value of the portfolio and $B$ denote the value of the reserve asset. If the value of portfolio $A$ is above its rolling EDD Floor (25) at all times, then $A$ and $B$ satisfy the following conditions:

$$\bar{D}_{t-p,t}(A) - \bar{D}_{t_0,t}(B) \leq \bar{D}_{t-p,t}(A) - \bar{D}_{t_{t-p,A}^{**},t}(B) \leq x \quad (26)$$

if $R_B(t_{t-p,A}^{**}, t) \leq 0 \Rightarrow \bar{D}_{(t-p)^+,t}(A) \leq x + \bar{D}_{t_{t-p,A}^{**},t}(B)$ (27)

if $R_B(t_{t-p,A}^{**}, t) > 0 \Rightarrow \bar{D}_{(t-p)^+,t}(A) \leq x$ (28)

for all $t \in [t_0, \infty)$.

The proof of the Proposition is presented in Appendix E.

Notice that the rolling EDD constraint (22) is equivalent to (26). Thus, the rolling EDD Floor (25) is less conservative than the global EDD Floor, in the sense that it insures a less binding performance constraint and thus is has a lower expected opportunity cost, i.e. higher allocation to $S$, everything else being equal.

### 3 Controlling Relative Drawdowns

While the absolute loss control motive discussed above would typically concern the performance of a broad asset allocation policy, active investment managers specialized in a given asset class (e.g. equities) are often measured in relative terms with respect to a benchmark portfolio (e.g. market cap-weighted index). Although benchmarks provide an effective tool
for investors to monitor and control the quality of decisions made by their investment managers, they might not always be efficient or optimal in the opinion of investment managers who aim at outperforming their benchmarks by taking different exposures to the available risk factors, either by means of security selection, optimization methods, market timing, or a combination of those.

A common practice to deal with this information asymmetry between manager and investor is to impose tracking error constraints on the standard deviation of the differences in periodic returns between the portfolio and the benchmark. From the standpoint of investors, perhaps a more sensible measure of relative risk is the maximum cumulative underperformance with respect to the benchmark. In fact, if a given manager accumulates too much underperformance with respect to the benchmark, in general there is no guarantee that the manager will be able to recover back the lost ground in terms of wealth with respect to its benchmark. Hence, limiting the benchmark underperformance at all times, can limit the potential regret of the investor in the long-run.

Indeed, severe underperformance relative to a given benchmark, such as standard equity market indices which are relatively cheap to replicate, is one of the main risks faced by portfolio managers and institutional investors seeking to outperform their benchmarks by investing in active investment funds with ‘alpha’ or ‘alternative betas’ strategies.

In order to measure this benchmark-relative risk, we introduce the maximum relative drawdown, which is a measure of the maximum cumulative underperformance of a portfolio relative to a given benchmark. In other words, the relative drawdown measures the maximum relative loss of a portfolio with respect to the given benchmark.

Let the relative value process of any given portfolio $A$ with respect to benchmark $B$ be denoted as $Z = \frac{A}{B}$. This is equivalent to a change of numeraire where the value of the portfolio is measured in shares of the benchmark asset instead of dollars.

The relative value $Z$ increases (decreases) when portfolio $A$ outperforms (underperforms) benchmark $B$. In fact, note that for log returns, $\log \left( \frac{Z(t)}{Z(s)} \right) = r_A(s, t) - r_B(s, t)$ for any $s \in [0, t]$ and $t \in [s, \infty)$. For instance, the process $\log \left( \frac{Z(t)}{Z(s)} \right)$ is defined as the relative return process by Fernholz (2002) (page 16). Thus, we define the relative drawdown as follows.

**Definition** ($\overline{RD}_{t_0, t}(A, B)$): The relative drawdown of portfolio $A$ with respect to the benchmark $B$ at time $s$ is defined as

$$RD_s(A, B) := D_s(Z),$$

and the maximum relative drawdown at time $t$ is defined as

$$\overline{RD}_{t_0, t}(A, B) := \overline{D}_{t_0, t}(Z),$$

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for $Z(s) = \frac{A(s)}{B(s)}$  \hspace{1cm} \forall s \in [t_0, t]$.

**Definition (RDD Floor):** Let the Relative Drawdown Floor value process for a type (1) strategy be defined as

$$F(t) = k \frac{A(t^*_0, Z)}{B(t^*_0, Z)} B(t),$$

for all $t \in [t_0, \infty)$, where $A$ is the value of the portfolio, $B$ the value of the benchmark, $Z := A/B$.

Notice that $t^*_0, Z$ in (29) is defined with respect to the ratio $A/B$, thus one recovers the discounted wealth Floor (12) when the numeraire is the savings account, i.e. $B = S_0$. In Appendix H, we show that this Floor is insurable using a type (1) dynamic asset allocation strategy, for any investable and liquid benchmark asset.

The following proposition shows that if the value of the portfolio is always above the RDD Floor (17), then its maximum relative drawdown is lower than the risk budget $x$. Additionally, it shows that the underperformance to the benchmark asset, measured as the maximum difference in log returns between any two times $s$ and $t$ such that $s \leq t$, is also limited to $\tilde{x} = -\log(1-x)$.

**Proposition 3** Let $A$ denote the value of a portfolio and $B$ denote the value of the benchmark asset. If the value of portfolio $A$ is always above the corresponding RDD Floor (29), then $A$ and $B$ satisfy the following conditions:

$$\RRD_{t_0,t}(A, B) \leq x$$

$$r_A(s, t) - r_B(s, t) \geq -\tilde{x}$$

for all $s \leq t$ and $t \in [s, \infty)$, where $\tilde{x} = -\log(1-x)$ and $r^X(s, t) = \log \left( \frac{X_t}{X_s} \right)$ for any $X > 0$.

The proof of the proposition is presented in Appendix F.

Similar to the drawdown control strategy and unlike the EDD strategy, the relative drawdown strategy has a maximum exposure to the performance-seeking asset equal to $\sup_{s \in [t_0, t]} \omega_S(s) = m \times x$, which is reached every time the portfolio’s relative value attains a new maximum.

**Remark 9** Similar to the MDD metric and Floor, the RDD Floor and risk metric can be defined for any given rolling period of time $p$, simply by setting $t_0 = (t - p)^+$. In Appendix H we show that such strategy with a constant look-back period is also insurable for any $p$. 

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**Corollary 3** Let the relative trailing performance of a portfolio $A$ be defined as the outperformance with respect to benchmark $B$ over a period $p$, i.e., $r_A(t-p,t) - r_B(t-p,t)$ for all $t \in (p, \infty)$. If the value of $A$ is always above the corresponding rolling RDD Floor, then it satisfies at all $t > p$,

$$r_A(t-p,t) - r_B(t-p,t) \geq -\tilde{x}$$

where $\tilde{x} = -\log(1-x)$. Thus, a rolling RDD strategy with risk budget $x$ can guarantee that the relative trailing performance of the portfolio maintains above $-\tilde{x}$ at all times. The proof of the corollary is provided in Appendix G.

**Remark 10** The RDD strategy insures a proportion $k$ of the past maximal gains in relative value made by the portfolio since time $t_0$. This follows from inequality (60),

$$\frac{Z(s)}{Z(t_0)} \geq k \frac{Z(s_{t_0,Z})}{Z(t_0)} \quad \forall s \in [t_0, t]$$

$$\Leftrightarrow 1 + R_Z(t_0, s) \geq k(1 + R_Z(t_0, s_{t_0,A}))$$

for all $t_0 \in [0, s]$ and $s \in [t_0, \infty)$. Thus the RDD Floor induces a ratchet effect on the portfolios’ relative value.

**Remark 11** A guarantee of a positive relative return over the investment period $t_0$ to $t$ for all $t \in [t_0, \infty)$ (and on the relative trailing performance for $t_0 = (t - p)^+$) can be offered to the investor, i.e., $r_A(t_0, t) - r_B(t_0, t) > 0$ whenever the maximal realized relative return of the portfolio over the (rolling) period attains the following level:

$$r_Z(t_0, t_{t_0,Z}^*) = r_A(t_0, t_{t_0,Z}^*) - r_B(t_0, t_{t_0,Z}^*) > \frac{x}{1-x}.$$  \hspace{1cm} (32)

This follows from inequality (60) (with $t_0 = (t - p)^+$ for the trailing performance).

## 4 Conclusion

In this article, we introduce and characterize several variations of the maximum drawdown control strategy that allow investors to 1) control absolute losses while using a reserve asset different from cash, 2) control absolute and relative trailing performance, and 3) control maximum underperformance with respect to an investable stochastic benchmark, limiting them to a maximum risk budget set ex-ante.

The risk management constraints of these portfolio insurance strategies are relevant for long-term investors seeking to control short-term absolute losses or maintaining benchmark
underperformance below a chosen tolerance level at all times. We show that these strategies have interesting properties, such as a (conditional) ratchet effect on wealth or cumulative benchmark outperformance, thus implying a strictly increasing lower bound on wealth (or relative value).

Unlike other portfolio insurance strategies, such as CPPI, all the strategies introduced in this paper can insure their risk management constraints for investors entering at any point in time to an open fund.

Appendices

A Upper bound of the multiplier

Hereafter we provide the necessary condition that the multiplier of the EDD and RDD strategies should satisfy to avoid a Floor trespassing or “gap event”, and hence to be able to insure its performance constraint in discrete-time trading. This condition is the same for the multiplier of the CPPI strategy, in its general case with a locally risky reserve asset.

Hereafter, \( t^* \) denotes the latest running maximum time record at time \( t \) for either the portfolio value \( A \) (for the EDD Floor), or the relative value \( Z \) (for the RDD Floor). The results stated with this somewhat ambiguous notation hold for both definitions of \( t^* \), i.e., \( t^* = t^*_{0,A} \) and \( t^* = t^*_{0,Z} \), respectively.

Notice that from the definition of the EDD and RDD Floors, for all \( t = t^* \),

\[
F(t^*) = kA(t^*)
\]

\[\Rightarrow \frac{dF(t^*)}{F(t^*)} = \frac{dA(t^*)}{A(t^*)} \tag{33}\]

Hence, no other condition is needed for \( A \) to remain above \( F \) assuming that \( A(s) \geq F(s) \) for all \( s < t^* \). On the other hand, for \( t \neq t^* \),

\[
F(t) = \tilde{k}B(t) \text{ where } \tilde{k} = k\frac{A(t^*)}{B(t^*)}
\]

\[\Rightarrow \frac{dF(t)}{F(t)} = \frac{dB(t)}{B(t)} \tag{34}\]

since \( \tilde{k} \) is constant until a new maximum is recorded (i.e., \( t = t^* \)).

Let the Cushion process of portfolio \( A \) be defined as \( C(t) = A(t) - F(t) \) for all \( t \in [0, \infty) \). If \( A(t) \geq F(t) \), then the Cushion process satisfies \( C(t) \geq 0 \) at all times. Using the fact that \( \frac{dF(t)}{F(t)} = \frac{dB(t)}{B(t)} \) for all \( t \neq t^* \) (equation (34)), the dynamics of the Cushion process can be
written as follows:

\[
dC_t = d(A_t - F_t) \\
= dA_t - dF_t \\
= A_t \left( \frac{m_t C_t}{A_t} dS_t + \left( 1 - \frac{m_t C_t}{A_t} \right) \frac{dB_t}{B_t} \right) - F_t \frac{dB_t}{B_t} \\
= m_t C_t \frac{dS_t}{S_t} + (1 - m_t) C_t \frac{dB_t}{B_t} \\
= C_t \left( m_t \frac{dS_t}{S_t} + (1 - m_t) \frac{dB_t}{B_t} \right)
\]

(35)

for all \( t \neq t^* \) (in the former derivation we used the notation \( x_t \) instead of \( x(t) \) for readability).

Denote \( \Delta \) any given discrete time step, and denote \( \Delta X(t) := X(t + \Delta) - X(t) \) for any process \( X \). A discretization of equation (35) shows that for the Cushion to remain positive between any two trading moments \( t \) and \( t + \Delta \), the multiplier has to satisfy the following condition:

\[
\frac{C(t + \Delta)}{C(t)} - 1 = m_t \frac{S(t + \Delta)}{S(t)} + (1 - m_t) \frac{B(t + \Delta)}{B(t)} - 1 > -1 \Leftrightarrow \frac{S(t + \Delta)}{S(t)} > \frac{(m_t - 1) B(t + \Delta)}{m_t B(t)}
\]

or equivalently

\[
m_t R_S(t, t + \Delta) + (1 - m_t) R_B(t, t + \Delta) \geq -1 \\
m_t (R_S(t, t + \Delta) - R_B(t, t + \Delta)) \geq -(1 + R_B(t, t + \Delta)).
\]

(36)

For \( (R_S(t, t + \Delta) - R_B(t, t + \Delta)) < 0 \), the inequality (36) gets inverted:

\[
m_t \leq \frac{-(1 + R_B(t, t + \Delta))}{R_S(t, t + \Delta) - R_B(t, t + \Delta)} := M_t;
\]

(37)

thus, the maximum value for the multiplier that can guarantee in general the Cushion’s positivity condition is (37), for every \( t \) and \( t + \Delta \) for which the condition \( (R_S(t, t + \Delta) - R_B(t, t + \Delta)) < 0 \) is satisfied.

\( M_t \) is not known at time \( t \), hence it has to be estimated. Studies on the multiplier of the CPPI strategy including Bertrand and Prigent (2002), Cont and Tankov (2009), Hamidi et al. (2009), Hamidi et al. (2008, 2009a); Hamidi, Maillet, and Prigent (2009b), and Ben Ameur and Prigent (2013) address the question of estimating the maximum multiplier or upper bound that allows that type (1) strategy to insure its performance constraint for a given confidence level under discrete-time trading (or discontinuous prices). These studies assume a risk-free reserve asset with constant rate of return. In that particular case in which the reserve asset is risk-free, its returns are “relatively small” (Bertrand and Prigent, 2002).
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2002) compared to the worst possible expected returns of the risky asset. For this reason, the upper bound of the multiplier (37) is usually reduced to:

\[ m_t \leq \frac{-1}{R_S(t, t+\Delta)} \]

(38)

for every \( t \) and \( t+\Delta \) such that \( R_S(t, t+\Delta) < 0 \). Under the constant interest rate assumption, the left tail of the risky asset is the only matter of concern to guarantee that the strategy complies with its risk-management objective.

Although former papers on the portfolio insurance multiplier do not address the right-tail risk of the reserve asset, equation (37) shows that, when the reserve asset is \( \textit{locally} \) risky, a sudden and significant increase in its value may also cause a Floor violation. In other words, the right tail of the distribution of the reserve asset returns is of critical importance for the estimation of the upper bound of the multiplier in applications with locally risky reserve assets (as the strategies studied in this paper).

A conservative estimate of the upper bound of the multiplier \( \mathcal{M}_t \) of type 1 allocation strategies is given by,

\[ \hat{m}_t = \frac{1}{LT_S(t) - RT_B(t)} \]

(39)

where \( LT_S(t) \) and \( RT_B(t) \) are left and right tail estimates of the (conditional) return distributions of asset \( S \) and \( B \) respectively. In other words, \( -LT_S(t) \) and \( RT_B(t) \) correspond to the minimum and maximum expected return on assets \( S \) and \( B \) respectively, from time \( t \) until the next possible reallocation time. Tail estimates can be for instance (conditional) Expected Shortfall of \( R_S \) and of \( -R_B \) respectively. Corollary 4 below, together with the assumption that \( RT_B(t) > 0 \) at all times, shows why \( \hat{m}_t \) in equation (39) is a conservative estimate of \( \mathcal{M}_t \), that is to say, an estimation of a lower limit of the bound.

**Corollary 4** Consider any pair of assets \((S, B)\) and any two trading instants \( t \) and \( t+1 \), \( t \in [0, \infty) \). The following is a lower bound of the corresponding multiplier upper bound \( \mathcal{M}_t \),

\[ \overline{m}_t = \begin{cases} 
\frac{-1}{R_S(t+\Delta) - R_B(t+\Delta)} & \text{if } R_B(t+\Delta) > 0 \\
\frac{-1}{R_S(t+\Delta)} & \text{if } R_B(t+\Delta) \leq 0 
\end{cases} \leq \mathcal{M}_t, \]

Proof. Recall \( \mathcal{M}_t \) is defined for \( t \) such that \( (R_S(t+\Delta) - R_B(t+\Delta)) < 0 \). For notational convenience, hereafter we suppress \((t+\Delta)\) after \( R_S \) and \( R_B \).

Case \( R_B > 0 \). Suppose

\[ \frac{-1}{R_S - R_B} \leq \frac{-(1 + R_B)}{R_S - R_B}, \]

which together with \( (R_S - R_B) < 0 \), implies:

\[ \frac{1}{R_S - R_B} \geq \frac{1 + R_B}{R_S - R_B} \]

\[ 1 \leq \frac{1 + R_B}{R_S - R_B} \]
which proves the first condition.

Case $R_B \leq 0$. Suppose

$$\frac{-1}{R_S} \leq \frac{- (1 + R_B)}{R_S - R_B}.$$  

Notice that if $R_B \leq 0$ and $(R_S - R_B) < 0$ then $R_S \leq 0$. Thus

$$1 \leq \frac{R_S (1 + R_B)}{R_S - R_B}$$

$$R_S - R_B \geq R_S + R_SR_B$$

$$-1 \leq R_S$$

which is true from the definition of a return of an asset. \qed

## B Golden Rule Sufficiency Proof

Assuming that the multiplier process satisfies the condition aforementioned, hereafter we determine sufficient conditions to prove that a Floor is insurable with a type (1) strategy, for the non-trivial cases in which $F(t_0) < A(t_0)$, which holds for all risk budget parameters satisfying $k < 1$ for the EDD and RDD Floors and their rolling version.

Denote $\Delta$ any given discrete time step, and denote $\Delta X(t) := X(t + \Delta) - X(t)$ for any process $X$.

First notice that, since $F(t_0) < A(t_0)$ by assumption, then,

$$\text{if } \frac{A(t_0 + \Delta)}{A(t_0)} \geq \frac{F(t_0 + \Delta)}{F(t_0)} \Rightarrow A(t_0 + \Delta) \geq F(t_0 + \Delta).$$

In general, for any two times $s$ and $t := s + \Delta$, such that $A(s) \geq F(s)$,

$$\text{if } \frac{A(t)}{A(s)} \geq \frac{F(t)}{F(s)} \Rightarrow A(t) \geq F(t).$$

Thus, we have proven by induction that, for all times at which $\frac{A(t)}{A(s)} \geq \frac{F(t)}{F(s)}$, no other condition is needed for $A$ to remain above the Floor value at time $t$. Furthermore, for all at times $t$ at which the condition $F(t) \leq A(t)$ follows directly from the definition of the Floor (second condition of the golden rule), no further check is needed.

Hence, we need to find sufficient conditions for all other times, that is, for all times at which,

$$\text{C.1. : } \frac{A(t)}{A(s)} < \frac{F(t)}{F(s)}. \quad (40)$$

Inequality (40) corresponds to C.1 in the first condition of the Golden Rule definition (equation 2).
Using the same induction argument as above, we first assume $A(s) \geq F(s)$ and then find sufficient conditions so that $A(t) \geq F(t)$, for $t > s$. From equation (1) we have that

$$ R_A(s, s + \Delta) = m_s \left(1 - \frac{F(s)}{A(s)}\right) R_S(s, s + \Delta) + \left(1 - m_s \left(1 - \frac{F(s)}{A(s)}\right)\right) R_B(s, s + \Delta), $$

$$ R_A(s, s + \Delta) = m_s \left(1 - \frac{F(s)}{A(s)}\right) (R_S(s, s + \Delta) - R_B(s, s + \Delta)) + R_B(s, s + \Delta). \quad (41) $$

Recall that for $(R_S(t, t+1) - R_B(t, t+1)) < 0$, inequality (37) implies $m \leq M$. Hence, replacing $M$ by $m$ in equation (41) implies,

$$ R_A(s, s + \Delta) \geq R_B(s, s + \Delta) - (1 + R_B(s, s + \Delta)) \left(1 - \frac{F(s)}{A(s)}\right), $$

$$ R_A(s, s + \Delta) \geq (1 + R_B(s, s + \Delta)) \frac{F(s)}{A(s)} - 1, $$

$$ \frac{A(s + \Delta)}{A(s)} \geq \frac{B(s + \Delta) F(s)}{B(s) A(s)}. \quad (42) $$

Assuming the first statement of the first condition in the G.R., i.e.,

$$ \frac{B(s + \Delta)}{B(s)} \geq \frac{F(s + \Delta)}{F(s)}, \quad (43) $$

together with inequality (42) implies,

$$ \frac{A(s + \Delta)}{A(s)} \geq \frac{B(s + \Delta) F(s)}{B(s) A(s)} \geq \frac{F(s + \Delta)}{F(s)} \frac{F(s)}{A(s)}, $$

$$ A(s + \Delta) \geq F(s + \Delta). \quad (44) $$

Hence, as $t := s + \Delta$, condition (43) is a sufficient condition to prove that $A(t) \geq F(t)$, if $(R_S(s, s + \Delta) - R_B(s, s + \Delta)) < 0$. On the other hand, if $(R_S(s, s + \Delta) - R_B(s, s + \Delta)) \geq 0$, from equation (41) it is straightforward to see that $\frac{A(s+\Delta)}{A(s)} \geq \frac{B(s+\Delta)}{B(s)}$. Thus, inequality (43) is a sufficient condition to prove that $A(t) \geq F(t)$ in general. Inequality (43) is the first statement of the Golden Rule for $t = s + \Delta$. Since $\Delta$ is an arbitrary time step, this completes the proof. \(\square\)

**C Proof of Proposition 1**

*Proof.* If the value of the portfolio is always above the EDD Floor (17), then

$$ A(s) \geq k \frac{A(s_{t_0,A}^*)}{B(s_{t_0,A}^*)} B(s) $$

$$ 1 + R_A(s_{t_0,A}^*, s) \geq k(1 + R_B(s_{t_0,A}^*, s)) $$

$$ R_A(s_{t_0,A}^*, s) - kR_B(s_{t_0,A}^*, s) \geq -x, \quad (45) $$

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for all $s \in [t_0, \infty)$. Inequality (45) implies conditions (19) and (20) of the proposition. To verify this, notice that from condition (45),

$$\text{if } R_B(s_{t_0,A}^*, s) \leq 0 \Rightarrow R_A(s_{t_0,A}^*, s) - R_B(s_{t_0,A}^*, s) \geq R_A(s_{t_0,A}^*, s) - kR_B(s_{t_0,A}^*, s) \geq -x \quad (46)$$

for all $s \in [t_0, \infty)$. Multiply by $-1$ in both sides of inequality (46) and apply the definition of the drawdown to get

$$D_s(A) + R_B(s_{t_0,A}^*, s) \leq x.$$  

By definition of the drawdown, if $R_B(s_{t_0,A}^*, s) \leq 0$ then $-R_B(s_{t_0,A}^*, s) \leq D_s(B) \leq \bar{D}_{t_0,t}(B)$ for all $s \in [t_0, \infty)$. Since this holds for all $s$ including $s$ such that $D_s(A) = \bar{D}_{t_0,t}(A)$, then condition (19) in the proposition follows.

On the other hand, condition (45) implies that,

$$\text{if } R_B(s_{t_0,A}^*, s) > 0 \Rightarrow R_A(s_{t_0,A}^*, s) \geq -x + kR_B(s_{t_0,A}^*, s) \geq -x \quad (48)$$

Condition (20) follows by multiplying by $-1$ on both sides of inequality (48) and the definition of the drawdown, $D_s(A) = -R_A(s_{t_0,A}^*, s)$. This condition holds even for $s$ such that $D_s(A) = \bar{D}_{t_0,t}(A)$. Condition (18) follows from condition (19) and inequality (47); hence the Proposition follows. $\square$

D  

Proof of Corollary 2

Proof. Assuming $A(t) \geq F(t)$, from the definition of the EDD Floor it follows that

$$A(t) \geq k A(t_{t_0,A}^*) \frac{B(t_{t_0,A}^*)}{B(t_0)} B(t) \quad \forall t \in [t_0, \infty). \quad (49)$$

Dividing inequality (49) by $A(t_0)$, the first condition of the corollary follows from the definition of simple return. The second condition of the corollary follows by taking logs on both sides of inequality (49) and the definition of log return. $\square$

E  

Proof of Proposition 2

Proof. If the value of portfolio $A$ is always above its rolling EDD Floor (25), then

$$A(s) \geq k A(s_{t_0,A}^*) \frac{B(s)}{B(s_{s-p,A}^*)} B(s)$$

$$1 + R_A(s_{s-p,A}^*, s) \geq k(1 + R_B(s_{s-p,A}^*, s))$$

$$R_A(s_{s-p,A}^*, s) - kR_B(s_{s-p,A}^*, s) \geq -x, \quad (50)$$

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for all \( s \in [t_0, \infty) \). Inequality (50) implies conditions (27) and (28) of the proposition. To verify this, notice that from condition (50),

\[
R_B(s_{s-p,A}^{**}, s) \leq 0 \Rightarrow R_A(s_{s-p,A}^{*}, s) - R_B(s_{s-p,A}^{**}, s) \geq R_A(s_{s-p,A}^{**}, s) - kR_B(s_{s-p,A}^{**}, s) \geq -x
\]

(51)

for all \( s \in [t_0, \infty) \). Multiply by \(-1\) in both sides of inequality (51) and apply the definition of the drawdown to get

\[
D_{s_{s-p,A}^{**}}(A) + R_B(s_{s-p,A}^{**}, s) \leq x,
\]

(52)

where \( D_{s_{s-p,A}^{**}}(A) \) denotes the drawdown of \( A \) observed at time \( s \) since time \( s_{s-p,A}^{**} \). By definition of \( s_{s-p,A}^{**} \), we have that \( D_{(s-p)^+,s}(A) \leq D_{s_{s-p,A}^{**}}(A) \), hence inequality (52) implies:

\[
D_{(s-p)^+,s}(A) + R_B(s_{s-p,A}^{**}, s) \leq x.
\]

(53)

By definition of the drawdown, if \( R_B(s_{s-p,A}^{**}, s) \leq 0 \) then \( -R_B(s_{s-p,A}^{**}, s) \leq D_{s_{s-p,A}^{**}}(B) \leq \bar{D}_{t_0,t}(B) \) for all \( s \in [t_0, t] \); therefore,

\[
D_{(s-p)^+,s}(A) - \bar{D}_{t_0,t}(B) \leq D_{s_{s-p,A}^{**}}(A) - D_{s_{s-p,A}^{**}}(B) \leq D_{s_{s-p,A}^{**}}(A) + R_B(s_{s-p,A}^{**}, s) \leq x,
\]

(54)

for all \( s \in [t_0, \infty) \). Since condition (54) holds for all \( s \) including \( s \) such that \( D_{(s-p)^+,s}(A) = \bar{D}_{(s-p)^+,s}(A) \), then condition (27) in the proposition follows.

On the other hand, condition (50) implies that,

\[
R_B(s_{s-p,A}^{**}, s) > 0 \Rightarrow R_A(s_{s-p,A}^{**}, s) \geq -x + kR_B(s_{s-p,A}^{**}, s) \geq -x
\]

(55)

By multiplying by \(-1\) on both sides of inequality (55), from the definition of the drawdown, \( D_{s_{s-p,A}^{**}}(A) = -R_A(s_{s-p,A}^{**}, s) \), and noticing that \( D_{(s-p)^+,s}(A) \leq D_{s_{s-p,A}^{**}}(A) \) we have

\[
D_{(s-p)^+,s}(A) \leq D_{s_{s-p,A}^{**}}(A) \leq x.
\]

(56)

Inequality (56) holds also for \( s \) such that \( D_{(s-p)^+,s}(A) = \bar{D}_{(s-p)^+,s}(A) \), thus condition (28) follows.

\[\square\]

**F  Proof of Proposition 3**

If the value of the portfolio is always above the RDD Floor (29), then the following statements hold:

\[
A(s) \geq k \frac{A(s_{t_0,Z}^*)}{B(s)} B(s)
\]

\[
\frac{A(s)}{B(s)} \geq k \frac{A(s_{t_0,Z}^*)}{B(s)}
\]

(57)

\[
\frac{Z(s)}{Z(s_{t_0,Z}^*)} - 1 \geq k - 1 = -x
\]
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for all \( s \in [t_0, \infty) \), which proves condition (30) in the proposition. Moving all terms except \( k \) to the left hand side of equation (57) and taking the logs, we obtain,

\[
\log \left( \frac{Z(s)}{Z(q)} \right) \geq \log \left( \frac{Z(s)}{Z(s^{\star}_{t_0,Z})} \right) \geq \log(1 - x),
\]

for all \( q \leq s \) and \( s \in [t_0, \infty) \). Noting that \( \log \left( \frac{Z(s)}{Z(q)} \right) = r_{A}(q,s) - r_{B}(q,s) \) for any \( q \in [0, s] \) and \( s \in [0, \infty) \) yields condition (31) in the proposition.

\[ \square \]

\textbf{G Proof of Corollary 3}

Divide inequality (59) below by \( Z(t_0) \) for \( t \in [p, \infty) \) and with \( t_0 = (t - p)^{+} \),

\[
A(t) \geq k \frac{A(t_0^{\star}_{t_0,Z})}{B(t_0^{\star}_{t_0,Z})} B(t)
\]

\[
Z(t) \geq Z(t_0) \geq k \frac{Z(t_0^{\star}_{t_0,Z})}{Z(t_0^{\star}_{t_0,Z})}.
\]

Taking logs in inequality (60) and noting that by definition of \( t_0^{\star}_{t_0,Z} \),

\[
r_{A}(t_0, t_0^{\star}_{t_0,Z}) - r_{B}(t_0, t_0^{\star}_{t_0,Z}) \geq 0,
\]

it follows that \( r_{A}(t_0, t) - r_{B}(t_0, t) \geq -\tilde{x} \) for all \( t \geq p \).

\[ \square \]

\textbf{H Proving the Insurability of EDD and RDD Floors}

In what follows, we show that both the RDD and EDD Floors definitions satisfy the golden rule (2) using continuous-time and discrete-time arguments. Unless indicated, in this section, \( t^{\star} \) denotes the latest running maximum time record at time \( t \) for either the portfolio value \( A \) (for the EDD Floor) or the relative value \( Z \) (for the RDD Floor). The results stated with this somewhat ambiguous notation hold for both definitions of \( t^{\star} \), i.e., \( t^{\star} = t_0^{\star}_{0,A} \) and \( t^{\star} = t_0^{\star}_{0,Z} \), respectively.

\textit{Continuous-time proof.}

Let \( t \in [0, \infty) \). Hereafter, we show that for all \( t \neq t^{\star} \), the return of the reserve asset is equal to the ‘return’ of the Floor (i.e., replication) and that for any \( t = t^{\star} \), condition C.1 in (2) does not hold (i.e., replication is not needed).

Notice that from the definition of the EDD and RDD Floors, for \( t = t^{\star} \),

\[
F(t^{\star}) = kA(t^{\star})
\]

\[
\Rightarrow \frac{dF_{t^{\star}}}{F_{t^{\star}}} = \frac{dA_{t^{\star}}}{A_{t^{\star}}}.
\]

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Hence, condition C.1 in the G.R. does not hold (i.e., no replication needed). On the other hand, for $t \neq t^*$,

$$F(t) = \tilde{k}B(t) \text{ where } \tilde{k} = \frac{A(t^*)}{B(t^*)}$$

$$\Rightarrow \frac{dF_t}{F_t} = \frac{dB_t}{B_t} \quad (62)$$

since $\tilde{k}$ is constant until a new maximum is recorded (i.e., $t = t^*$).

\[\square\]

Discrete-time proof.

Consider any two times, $s < t$, $s \in [0, t)$ and $t \in [0, \infty)$ and their corresponding running maximum time records $s^* \leq t^*$.

First we show that for all $s$ and $t$ such that $s^* = s = t^* < t$ or such that $s^* = t^* < s < t$, the reserve asset replicates the Floor. Second, we consider the cases $s^* = s < t^* = t$ and $s^* = s < t^* = t$ and show that no Floor replication is needed, as by definition $A_{t^*} > F_{t^*}$ for $0 < k < 1$ (which are the only relevant values for $k$). All other well defined possibilities for $s$ and $t$ are covered with combinations of the former cases\(^{11}\).

Cases $s^* = s = t^* < t$ and $s^* = t^* < s < t$

Replacing $A(t^*)$ and $B(t^*)$ with $t^*$ in the Floor formula yields

$$\frac{F(t)}{F(s)} = \frac{B(t)}{B(s)}; \quad (63)$$

hence the reserve asset $B$ replicates the Floor $F$.

Cases $s^* < s < t^* = t$ and $s^* < t^* = t$

From the definitions of the Floors and of $s^*$, it follows that

$$\frac{F(s)}{F(s^*)} = \frac{B(s)}{B(s^*)}. \quad (64)$$

Replacing $A(t)$ and $B(t)$ with $t = t^*$ in the Floor formula and using result (64), it follows that

$$\frac{F(t^*)}{F(s^*)} = \frac{kA(t^*)}{kA(s^*)} \frac{B(s)}{B(s^*)} = \frac{A(t^*)B(s^*)}{A(s^*)B(s)} \frac{F(t^*)}{F(s^*)} \Rightarrow A(t^*) = \frac{F(t^*)}{F(s^*)}, \quad (65)$$

which proves that for the case $s = s^* < t^* = t$, no Floor replication is needed. Furthermore, by definition of $F$, for $t = t^*$,

$$F(t^*) = kA(t^*) < A(t^*) \text{ for } k < 1.$$ 

Hence, the Floor cannot be violated for any case in which $t = t^*$, by the very definition of the Floor (second condition in the G.R.). As the arguments above hold for every $t^*$, this completes the proof. \[\square\]

\(^{11}\)Notice that the case $s^* < t^* < s < t$ is not possible for $t_0 = 0$ or any other constant value of $t_0 < s$. 

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Note that the EDD and RDD Floors can be defined for a rolling look-back period $p$ by setting $t_0 = (t - p)^+$. The golden rule check above holds for all the cases aforementioned in the discrete-time proof by setting $t_0 = (t - p)^+$.

However, the discrete time proof of the golden rule above does not include the case $s^* < t^* < s < t$ because by definition of $t^*$, for $t_0 = 0$ (or any other constant value of $t_0 < s$), that case is not possible. To see this, notice that by definition of $t^*$, if $t_0 = 0$, then $A(t^*_{t_0,A}) \geq A(s^*_{t_0,A})$ and $Z(t^*_{t_0,A}) \geq Z(s^*_{t_0,A})$. Hereafter, we also consider the case in which $A(t^*_{(t-p)^+,A}) < A(s^*_{(t-p)^+,A})$ and $Z(t^*_{(t-p)^+,A}) < Z(s^*_{(t-p)^+,A})$, which is only relevant for the rolling period definitions of EDD and RDD Floors, respectively.

**H.1 Golden rule check for rolling RDD Floor**

For any two times $s$ and $t$ such that $s^* < t^* < s < t$ we have,

$\frac{F(t)}{F(s)} = \frac{A(t^*_{(t-p)^+,Z}) B(s^*_{(s-p)^+,Z})}{A(s^*_{(s-p)^+,Z}) B(t^*_{(t-p)^+,Z})} \frac{B(t)}{B(s)}. \quad (66)$

According to the golden rule (2), if $\frac{A(t)}{F(s)} < \frac{F(t)}{F(s)}$, then the reserve asset should super-replicate the Floor index, i.e., $\frac{B(t)}{B(s)} \geq \frac{F(t)}{F(s)}$.

For the opposite condition to hold,

$\frac{F(t)}{F(s)} = \frac{A(t^*_{(t-p)^+,Z}) B(s^*_{(s-p)^+,Z})}{A(s^*_{(s-p)^+,Z}) B(t^*_{(t-p)^+,Z})} \frac{B(t)}{B(s)} \geq \frac{B(t)}{B(s)},$

then the following must hold:

$\frac{Z(t^*_{(t-p)^+,Z}) B(t)}{Z(s^*_{(s-p)^+,Z}) B(s)} \geq \frac{B(t)}{B(s)},$

Thus the golden rule cannot be violated in any case in which $\frac{Z(t^*_{(t-p)^+,Z}) B(t)}{Z(s^*_{(s-p)^+,Z}) B(s)} > 1$.

Thus the golden rule cannot be violated in any case in which $Z(t^*_{(t-p)^+,Z}) < Z(s^*_{(s-p)^+,Z})$, which is the only case not covered on the proof of the global EDD and RDD floors.

**H.2 Golden rule check for rolling EDD Floor**

Hereafter we first show that the naive version of the rolling EDD Floor does not comply with the golden rule in a particular case under certain conditions provided below. The definition of the rolling EDD Floor version follows directly from those conditions.

Consider the case in which $A(t^*_{(t-p)^+,A}) < A(s^*_{(t-p)^+,A})$, which is only possible for the rolling period definitions of the EDD constraint and Floor.
For the golden rule to be violated, the following two conditions must hold at the same time:

\[
\frac{F(t)}{F(s)} = \frac{A(t_{(t-p)}^+,A)}{A(s_{(s-p)}^+,A)} \frac{B(s_{(s-p)}^+,A)}{B(t_{(t-p)}^+,A)} \frac{B(t)}{B(s)} > 1. \tag{67}
\]

Condition (67) is equivalent to the following condition:

\[
\frac{A(t_{(t-p)}^+,A)}{A(s_{(s-p)}^+,A)} > \frac{B(t_{(t-p)}^+,A)}{B(s_{(s-p)}^+,A)}. \tag{69}
\]

Notice that by assumption

\[
\frac{A(t_{(t-p)}^+,A)}{A(s_{(s-p)}^+,A)} < 1. \tag{70}
\]

Condition (70) together with condition (69), implies that the following condition must hold as well:

\[
\frac{B(t_{(t-p)}^+,A)}{B(s_{(s-p)}^+,A)} < 1. \tag{71}
\]

Hence, the naive EDD Floor with a rolling horizon does not respect the golden rule in the particular case in which the following four conditions hold at the same time:

\[
A(t_{(t-p)}^+,A) < A(s_{(s-p)}^+,A) \tag{71}
\]

\[
B(t_{(t-p)}^+,A) < B(s_{(s-p)}^+,A) \tag{72}
\]

\[
\frac{B(t_{(t-p)}^+,A)}{B(s_{(s-p)}^+,A)} < \frac{A(t_{(t-p)}^+,A)}{A(s_{(s-p)}^+,A)} \tag{72}
\]

\[
\frac{A(t)}{A(s)} < Q \frac{B(t)}{B(s)},
\]

where \( Q := \frac{A(t_{(t-p)}^+,A)}{A(s_{(s-p)}^+,A)} \frac{B(s_{(s-p)}^+,A)}{B(t_{(t-p)}^+,A)} > 1 \) (notice that for a fixed \( t_0 \), condition 71 cannot hold).

Recall that the golden rule is a sufficient although not necessary condition for a Floor to be insurable. Hence, if a strategy does not respect the golden rule, it does not necessarily mean that its Floor is not insurable. However, in Monte Carlo simulations, we did find some trespassing of this naive version of the rolling EDD Floor. Condition (72) can be restated as

\[
\frac{Z(t_{(t-p)}^+,A)}{Z(s_{(s-p)}^+,A)} > 1. \tag{73}
\]

Define the set

\[
\mathcal{G}(t) = \left\{ t_0 \leq s < t : \frac{Z(t_{(t-p)}^+,A)}{Z(s_{(s-p)}^+,A)} > 1 \right\}. \tag{74}
\]
which contains all times at which condition (73) is satisfied. Using the notation of
the rolling running maximum, i.e., \( M_{t-p}(t) := A((t_{t-p})^+,A) \), condition (71) can be restated as
\[
M_{t-p}(t) < M_{s-p}(s). \tag{75}
\]
Hence, the naive version of the EDD Floor does not respect the golden rule when conditions
(74) and (75) hold at the same time. We now define a conditional version of the rolling
EDD Floor in which the oldest date considered to determine the rolling running maximum
does not always move forward in time continuously, i.e. \( t_0 = t - p \) for all \( t > p \), but only
at times at which conditions (74) and (75) are not satisfied. This implies a conditional
version of \( t^*_A \), which is equal to its unconditional version \( t^*_A \) except when (74) and (75)
hold. For this conditional version, before re-setting the oldest date \( t_0 \) that determines the
rolling running maximum, to \( t_0 + \Delta t \), we check whether the conditions are satisfied; if the
conditions hold, then \( t_0 \) is not updated. Formally, let the conditional rolling maximum time
process be defined as
\[
t^{**}_{t-p,A} = \begin{cases} 
\sup_{s \in \mathcal{G}(t)} \left\{ s^*_{(s-p)^+,A} \right\}, & \text{if } t > t_0, \quad M_{t-p}(t) < M_{s-p}(s) \text{ and } \mathcal{G}(t) \neq \emptyset \\
(76)
t^*_{(t-p)^+,A}, & \text{otherwise.}
\end{cases}
\]
The definition of the rolling EDD is similar to its naive version, with the exception that
its reference time \( t^{**}_{t-p,A} \) for the running maximum of \( A \) only rolls forward if conditions (74)
and (75) are satisfied, and is equal to its preceding reference time otherwise. This modified
or conditional version of the rolling EDD Floor respects the golden rule by construction,
therefore it is insurable. This result was confirmed with Monte Carlo simulations (available
upon request from the author).

Hereafter we present an algorithm with a function that illustrates the definition of the
(conditional) rolling EDD Floor.
Algorithm Rolling EDD Floor

1: $t \leftarrow 1$ \hspace{1cm} \triangleright \text{Initialization}
2: $A^* \leftarrow 0$
3: $B^* \leftarrow 0$
4: $Z^* \leftarrow 1$
5: while $t \leq T$ do
6: \hspace{0.5cm} $[F(t), A^*, B^*, Z^*] \leftarrow \text{EDD_Floor}(A(1 : t), B(1 : t), Z(1 : t), A^*, B^*, Z^*, t, p)$
7: \hspace{0.5cm} $A^* \leftarrow A^*$
8: \hspace{0.5cm} $B^* \leftarrow B^*$
9: \hspace{0.5cm} $Z^* \leftarrow Z^*$
10: \hspace{0.5cm} $t \leftarrow t + 1$
11: end while
12: function EDD_Floor$(A(1 : t), B(1 : t), Z(1 : t), A^*, B^*, Z^*, t, p)$
13: \hspace{0.5cm} $t_0 \leftarrow \max\{t - p, 1\}$
14: \hspace{0.5cm} $A^* \leftarrow \max\{A(t_0 : t)\}$
15: \hspace{0.5cm} $t^* \leftarrow \text{find}(A(t_0 : t) == A^*, \text{last})$ \hspace{0.5cm} \triangleright \text{Find index of the last element equal to } A^* \text{ in } A(t_0 : t) \text{ array}
16: \hspace{0.5cm} $B_{aux} \leftarrow B(t_0 : t)$
17: \hspace{0.5cm} $B^* \leftarrow B_{aux}(t^*)$
18: \hspace{0.5cm} $Z_{aux} \leftarrow Z(t_0 : t)$
19: \hspace{0.5cm} $Z^* \leftarrow Z_{aux}(t^*)$
20: \hspace{0.5cm} if $(Z^*/Z^*) > 1 \& A^* < A^*$ then \hspace{0.5cm} \triangleright \text{Conditions for G.R. exception}
21: \hspace{1.0cm} $F(t) \leftarrow k \times A^* \times (B(t)/B^*)$
22: \hspace{1.0cm} $A^* \leftarrow A^*$
23: \hspace{1.0cm} $B^* \leftarrow B^*$
24: \hspace{1.0cm} $Z^* \leftarrow Z^*$
25: \hspace{0.5cm} else
26: \hspace{1.0cm} $F(t) \leftarrow k \times A^* \times (B(t)/B^*)$
27: \hspace{0.5cm} end if
28: \hspace{0.5cm} return $F(t), A^*, B^*, Z^*$
29: end function

References


Guasoni, P. and J. Obloj (2011). The incentives of hedge fund fees and high-water marks.


